Application of Multivariate Garch in Modeling the Returns on Exchange Rate and Crude Oil Prices in Nigerian Foreign Market

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Abstract  
This study examines the application of multivariate Generalized Autogressive Conditional Heteroscedasticity (MGARCH) in modeling the returns on exchange rate and crude oil prices in Nigerian foreign markets. The data on Naira/Dollar exchange rate and crude oil prices used in this study was sourced for and extracted from the central bank of Nigeria (CBN) online statistical database within the period 1991 to 2017. The model were estimated using Eview software version ten (10). However, before the estimation of the model preliminary time series checked were done on the data and the results revealed the present of volatility clustering. Results reveal the estimate of the maximum lag for exchange rate to be 3 and crude oil prices 2 respectively. Also, the results confirmed that there are two cointegrating equations in the relationship between the returns on exchange rate and crude oil prices. The coefficient associated with the Error Correction Model (ECM) for the returns on exchange rate - 0.314107) and significant at 10% level of significant while crude oil prices was (-0.627834) but not significant. The coefficient of the two Error Correction Model (ECM) were negative. This result shows that at least 31.41% and 62.7834% of the co-movements into disequilibrium between the returns on exchange rate and crude oil prices are corrected within one period. The results of the MGARCH estimation also confirmed the presence of two directional volatility spillovers between the two foreign markets.

Keywords: Volatility, Dynamics GARCH, Exchange Rate, Crude Oil Prices.

1.1 Introduction  
Nigeria just like any other country in the world is engaged in trade with other countries sometimes on the ground of economists’ concept and the doctrine of comparative advantage. A doctrine which encourage countries in the world to produce goods and services in which they are endowed with in order to maximize the cost of producing such goods and services, and maximizes profit. This situation is not far fetch as Nigeria is engaged in several forms of foreign trade, for example exporting crude from Nigeria to other countries that do not produce crude oil as their economic mainstay. They also exchange their naira for other currencies for the purposes of earning a living or basically for economic transaction. In the process of trading in terms of exporting crude oil for sales and exchanging their naira for other currencies, there are associated challenges in the form of dynamic of price volatility. These challenges could be cause by sharp practices by the traders, drop in crude oil prices, economic disaster and sabotage like uprising in Oil producing areas etc. All these are the major causes of the dynamic of volatility experience on the returns on exchange rate and crude oil prices. According to Oloyede...
and Essi (2017), drop in oil prices in the world market has led to drastic reduction in foreign earnings with its corresponding consequences on other macro economic variables. Therefore, the depreciation of local currency in foreign market has also affected the non-oil sector and imported goods become expensive. Oloyede and Essi (2017) further suggested that economists believe that appreciation of exchange rate increases imports, conversely, depreciation would increase exports and discourages imports.

Similarly, volatility in the returns of exchange rates and crude oil prices are nowadays a common economic phenomenon which cannot be under-estimated. This is because exchange rates and crude oil prices are highly connected to each other such that one depends on the other. According to Liang and Minh (2012) given the fact that oil is quoted in U.S. dollars, it is natural to suggest a way to explain that exchange rates drive oil prices. They further suggested that all things been equal, when the U.S. dollar depreciates, conversely, oil – exporting countries would increase oil prices in order to stabilize the purchasing power of their currencies export and revenues in terms of their (predominantly euro – denominated) imports. This has direct effect on the reduction in supply curve which bring about leftward shift in the supply curve. Conversely, in the case of other commodities, foreign exchange rate depreciation makes crude oil less valuable for consumers in other countries (in local currency); and consequently leads to increasing demand for crude oil.

Sequel to the above facts, both its resultant consequence and the supply increase in demand, cause an increase in the returns on exchange rate and crude oil prices denominated in foreign currency (U.S dollar).

Also, it is well known that crude oil prices are dominated in U.S dollars, and so volatility in domestic currencies (exchange rates) depends closely on the dollar exchanges, oil and non- oil economics are not affected by the oil prices increase in the same manner (Veysel and Caneer 2018). According to Liang and Minh (2012), high fluctuation on exchange rates and crude oil markets result in a more challenging trade execution, exposing traders in foreign markets to risks, huge capital requirement to invest into their business and decreasing the effectiveness of benchmark hedging relatively compare to other asset classes. This is because exchange rates are seen to be determined by expected future fundamental conditions, among which crude oil is surely an important key player as supported by empirical evidence found in Yousefi and Wirjanto (2004), Krichene (2005), Zhang, fan, Isai and Wee (2008), and Leang and Minh (2012). Increase in the returns on crude oil prices lead to stronger economies for oil- exporters and higher production costs for oil- exporters. Hence, it may leads to appreciation of oil-exporter currencies comparatively to those of oil-importers. Consequently, it is likely that the causality between these two economic indicators runs from oil prices to foreign exchange rate (Liang and Minh, 2012). Krugman (1984) chandhuri and Aamiel (1998), Ayadi (2005), Chen and Chen (2007), coundert, Mignon and Penot (2007), Ben assy – Querea Migmonb and Penot (2007) have all provided evidence supporting this view. All these studies (irrespective of their mixed implications and contribution to knowledge) tend to suggest that crude oil prices and exchange rates most likely both contain information that can affect each other negatively or positively.

However, besides this study examines the dynamic of the co-movement and the inter-relationship between these variables, their associated volatility price stability discovery and risk transfer were considered crucial contributions of future market towards the arrangement of economic activity. According to Kaneham et al (2017) price discovery could be defined as the use of future prices for pricing cash market transactions. It is simply referred to as a way in which futures (return) price serves as market expectations of subsequent prices while the
stabilization of the return on exchange rate and crude oil prices are more important than their
dynamites since volatility makes it difficult for the major players and the countries. Exchange
rates dynamics has a way to influenced economic activity, equity markets and strategies used
by oil producing countries.
Therefore, instead of investigating the relationship between exchange rates and crude oil price
which many other studies cited above have done, this present study examines the dynamic
nature of volatilities on the returns on exchange rates and crude oil prices in Nigerian foreign
markets. Secondly, this study attempts to extract necessary information intertwined in the two
market, its associated risks in an error correction multivariate GARCH modeling process.

Methodology
The estimation procedure for all models specified above starts with the following steps:

i. **ARMA Model Estimation:** This is done by obtaining the residuals first from the
    ordinary least squares regression of the conditional mean equation which might be
    represented in (ARMA) process. Supposing an ARMA (1,1) process is considered for
    example, if the conditional mean equation is given as thus:

\[ X_t = \theta_1 X_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \]  

Where \( X_t \) represents the series of the variables to be fitted in the model

ii. **Descriptive Test Statistic for Normality Test**
The normality test is carried out using the Jarque-Bera test statistics. According to (Chinyere,
    Dickko & Isah, 2015) Jarque-Bera is defined as joint test of skewness and kurtosis that examine
    whether data series exhibit normal distribution or not; and this test statistic was developed by
    Jargue and Bera (1980). It is defined as:

\[ X^2 = \frac{N}{6} \left[ S^2 + \frac{(K-3)^2}{4} \right] \]  

Where \( S \) represents Skewness, \( K \) represents Kurtosis and \( N \) represents the size of the
macroeconomic variables used. The test statistic under the Null hypothesis of a normal
distribution has a degree of freedom 2. When a distribution does not obey the normality test,
Abdulkarem et al, (2017) suggested that the alternative inferential statistic was to use GARCH
with its error distribution assumptions with fixed degree of freedom.

(iii) **Test for Co-integration**
There is need to identify the co-integrating relationship between the two variables series and
the two likelihood ratio tests to be used are the \( \lambda_{\text{Trace}} \) and \( \lambda_{\text{Max}} \) respectively.

\[ \lambda_{\text{Trace}} = -T \sum_{t=r+1}^{n} \ln (1 - \lambda_t) \]  

For \( i = 0, 1, \ldots, n - 1 \)

\[ \lambda_{\text{Max}} = -T \ln (1 - \lambda r + 1) \]  

Where \( n \) is the number of usable observations and \( \lambda_t \) are the estimated eigen values otherwise
refers to as characteristics roots, the trace test statistic (\( \lambda_{\text{Trace}} \)) test the null hypothesis of \( r \) co-
integrating relationship Vs the alternative hypothesis of less than or equal to \( r \) co-integrating
relationship. Similarly, \( \lambda_{\text{Max}} \) test statistic examines the null hypothesis of \( r \) co-
integrating relation against \( r +1 \) co-integrating relations. However, the rank of \( \pi \) estimate can be
determined using \( \lambda_{\text{Trace}} \) or \( \lambda_{\text{Max}} \) test statistic. This is done on the condition that if rank of \( \pi = 1 \),
then there is single co-integrating vector and the estimator \( \pi \) can be factorized as \( \pi = a \beta \),
where $\alpha$ and $\beta$ are $\alpha \times 1$, vectors representing error correction co-efficient examining the speed of convergence and integrating parameters respectively.

iv. **Vector Error Correction Model:** The vector Error correction model (VECM) is used to investigate the causal relationship between the return on exchange rates and crude oil prices after identifying the appropriate order of integration of each variable. This is done by first identifying the significant lag length of the VAR model using suitable information criteria. If the returns on Nigerian/American exchange rate and crude oil prices series are co-integrated we can estimate the VAR model including a variable representing the deviations from the long-run equilibrium. The VECM model for variables including; constant, the error correction term and lagged form;

\[
\begin{align*}
\Delta s_t^{RE} & = [c_1 + d_1] + b_{11} b_{12} \Delta s_{t-1} + b_{21} b_{22} \Delta s_{t-1}^{RE} + e_{t}^{RE} \\
\Delta s_t^{RC} & = [c_2 + a_1] + ECT_{t-1} + [b_{11} b_{12}] \Delta s_{t-1} + [b_{21} b_{22}] \Delta s_{t-1}^{RE} + e_{t}^{RC}
\end{align*}
\]

(3.6)

RE represents returns on exchange rate, RC represent the returns crude oil prices series. The VECM estimation as a preliminary stage to model estimation is particularly necessary and interesting as it allows for estimation of how the variables adjust deviations towards the long-run equilibrium. The error correction co-efficient ($a_1$) reflects the speed of Adjustment.

**VECH–GARCH**

The multivariate extension of the univariate GARCH (p,q) called VECH–GARCH (P,Q) process could be defined as thus: Let $(\eta_t)$ be a sequence of identically independent distribution (iid) variables with distribution $\eta$. The process $(\varepsilon_t)$ is said to admit a VECH–GARCH (p,q) representation (relative to the sequence $(\eta_t)$) if it satisfies:

\[
\varepsilon_t = H_{t/2} \eta_t
\]

Where $H_t$ is positive definite such that

\[
Vech(H_t) = w + \sum_{j=1}^{N} A^{(i)} Vech \left( \Sigma_{j,t}, \Sigma_{r,s} \right) + \sum_{j=1}^{N} \beta^{(i)} \text{ vech} \left( H_{t,j} \right)
\]

Where $\omega$ is a vector of size $\left\{ \frac{N(NH)}{2} \right\} X 1$, and $A^{(i)}$ and $B^{(i)}$ are matrices of dimension

\[
\frac{N(NH)}{2} \times \frac{N(NH)}{2} A^{(i)} B^{(i)}: \left[ NXN \right], \text{ where N=2, VECH–GARCH (1,1)}:
\]

\[
\begin{bmatrix}
\begin{array}{c}
h_{11,t} \\
h_{21,t} \\
h_{22,t}
\end{array}
\end{bmatrix} =
\begin{bmatrix}
w_{1}^* \\
w_{2}^* \\
w_{3}^*
\end{bmatrix} +
\begin{bmatrix}
a_{11}^* a_{12}^* a_{13}^* \\
a_{21}^* a_{22}^* a_{23}^* \\
a_{31}^* a_{32}^* a_{33}^*
\end{bmatrix} \begin{bmatrix}
\Sigma_{1}^2 t^{-1} \\
\Sigma_{1,t-1}^2 \Sigma_{2,t-1}^2 t^{-1} \\
\Sigma_{2}^2 t^{-1}
\end{bmatrix}
+ \begin{bmatrix}
b_{19}^* b_{12}^* b_{13}^* \\
b_{21}^* b_{22}^* b_{23}^* \\
b_{31}^* b_{32}^* b_{33}^*
\end{bmatrix} \begin{bmatrix}
h_{11,t-1} \\
h_{21,t-1} \\
h_{22,t-1}
\end{bmatrix}
\]

(3.9)

where the number of parameters is given as $1 + (p+q) \left[ \frac{N(NH)}{2} \right]$, even for low dimensions of N and small values of p and q the number of parameters is very large, for N = 5 and p = q = 1 the unrestricted version of (1) which contains 465 parameters. Similarly, another estimator that models the conditional correlation was the Dynamic conditional correlation.

**Dynamic Conditional Correlation (DCC) Model**

According to Engle (2002), the Dynamic conditional correlation model process can be expressed in the following manner.
\[ H_t = D_t R_t D_t = P_{ijt} \frac{h_{ijt}}{h_{iit}} \]  

(3.10)

Where \( H_t \) conditional variance Co-variance matrix \( R_t \) is an nxn conditional correlation matrix. In this case, matrices \( D_t \) and \( R_t \) are estimated as follows:

\[ D_t = \text{diag} \left( h_{11}^{1/2}, \ldots, h_{ni}^{1/2} \right) \]  

(3.11)

Where \( h_{iit} \) is chosen to be a univariate GARCH (1,1) model, \( R_t = (\text{diag} \, Q_t) - \frac{1}{2} \) \( Q_t (\text{diag} \, Q_t)^{-\frac{1}{2}} \)

(3.12)

Where \( Q_t = \left( 1 - \alpha - \beta \right) Q + \alpha - \mu_{t-1} + \beta Q_{t-1} \) refers to an nxn symmetric positive definite matrix with \( \mu_n = \frac{e_{it}}{h_{iit}} \), \( Q \) is the nxn unconditional variance matrix of \( \mu_t \) and \( \alpha \) and \( \beta \) are non-negative scale parameter satisfying \( \alpha + \beta \leq 1 \). The conditional correlation co-efficient \( \ell_{ij} \) between two markets I and J are than computed as follows:

\[ \rho_{ij} = \frac{P_{ij}}{\left(1 - \alpha - \beta \right)Q_{ij} + \alpha \mu_i \cdot t - 1 + \beta \alpha_j \cdot t - 1}{\left(1 - \alpha - \beta \right)Q_{ij} + \alpha U_{ijt} + \beta Q_{ijt} + \beta q^2_{ijt}}^{\frac{1}{2}} \]  

(3.13)

Where \( P_{ij} \) represent the element located in the ith row and jth column of the symmetric positive definite matrix \( Q_t \).

**BEKK – GARCH Model**

The acronym BEKK simply represents Baba, Engle, Kraft and Kroner, which is a preliminary version of Engle and Kroner (1995). For a single series, the volatility pattern follow univariate specification of GARCH model of the form:

\[ h_t = c_o + a_1 e_{i-1}^2 + \ldots + a_p e_{i-p}^2 + b_1 h_{i-1} + \ldots + b_q h_{i-q} \]  

(3.14)

Where \( \rho \) and \( q \) are order of the GARCH Model. This can be generalized in the form

\[ H_t = C_o^t + \sum_{k=1}^{p} \sum_{i=1}^{q} \beta_{ik} H_{t-c}^{i} \beta_{i-k}^{t} \]  

(3.15)

Where \( C, A_k \) and \( \beta_{ik} \) are (nxn) matrix, \( C_o^t \) is the intercept of the matrix in a new posed form, where \( C \) is a lower triangular matrix and it is positive semi definite.

For BEKK (1, 1), it is represented as thus:

\[ H_t = C_o^t + A_1 e_{i-1} e_{i-1} + B_1^t H_{t-1} B_1 \]  

(3.16)

Where, \( A_1 \) and \( B_1 \) are nxn parameter matrix and \( C_o \) is nxn upper triangular matrix. Then, the Bivariate BEKK (1,1) model can be written as:

\[ H_t = C_o^t + A_{i1} e_{i1} + A_{i2} e_{i1} e_{i1} + B_{i1} H_{t-1} + B_{i2} \]  

(3.17)

The off diagonal parameter in matrix \( B, B_{12} \) and \( B_{21} \) respectively estimated as the independence of conditional volatility of the returns on exchange rate and crude oil price series. The \( b_{11} \) and \( b_{22} \) represents persistence in one set of variable of the returns on exchange rate and crude oil price series. Similarly, the parameter \( a_{12} \) or \( a_{21} \) represents the cross variable effects. Conversely, \( a_{11} \) and \( a_{22} \) represents the returns own effects.

Sequel to the above facts, the significant level of each parameter the presence of strong ARCH or GARCH effect, however, from the equation(3.17), We can deduce the following equations of conditional variance and conditional covariance:

\[ h_{11} = C_1 + a_{11}^2 e_{i1}^2 + a_{11} a_{21} e_{i1} e_{i1} + a_{21}^2 e_{i1} e_{i1} t - 1 + b_{11}^2 h_{11}^2 + 2 b_{11} b_{21} h_{11} h_{12} t - 1 + b_{21}^2 h_{21}^2 \]  

(3.18)
\[
\begin{align*}
\text{h}_{22t} &= C_3 + a_{12}^2 \varepsilon_{t-1}^2 + 2a_{12} a_{22} \varepsilon_{t-1} \varepsilon_{t} + a_{22}^2 \varepsilon_{t}^2 + b_{12}^2 h_{1t-1} + 2b_{12} b_{22} h_{1t-1} + b_{22}^2 h_{2t-1} \\
\text{h}_{12t} &= c_2 + a_{11} a_{12} \varepsilon_{t-1}^2 + (a_{21} a_{12} + a_{11} a_{22}) \varepsilon_{t-1} \varepsilon_{t} + a_{22}^2 \varepsilon_{t}^2 + b_{11} b_{12} h_{1t-1}^2 + b_{11} b_{22} h_{2t-1}^2 + (b_{21} b_{12} + b_{11} b_{22}) h_{1t-1} + b_{21} b_{22} h_{2t-1} \\
&\quad + (b_{21} b_{12} + b_{11} b_{22}) h_{1t-1} + b_{21} b_{22} h_{2t-1} \\
\end{align*}
\]

(3.19)

(3.20)

Estimation of Multivariate GARCH Models

According to Wenjing and Yiyu (2010), the most common method of estimating conditional covariance matrix in a Multivariate GARCH model is by the use of quasi maximum likelihood method. Assuming \( H_t(\theta) \) is a positive definite \( N \times N \) conditional covariance matrix of some \( N \times 1 \) residual vector \( \varepsilon_t \), parameterized by the vector \( \theta \). Denoting the available information at time \( t \) by \( \varepsilon_t \), we have:

\[
\varepsilon_{t-1} [\varepsilon_t, f_{t-1}] = 0;
\]

(3.21)

Generally, the conditional covariance matrix \( H_t(\theta) \) is well specified based on a certain MGARCH model. Suppose there is an underlying parameter vector \( \theta_0 \) which one wants to estimate using a given sample of \( T \) observations. The quasi maximum likelihood (QML) approach estimates \( \theta_0 \) by maximizing the Gaussian log likelihood

\[
\log L_t(\theta) = -\frac{NT}{2} \log (2\Pi) - \frac{1}{2} \sum_{t=1}^{T} \log H_t^{-1} - \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t^2 H_t^{-1} \varepsilon_t
\]

(2.23)

This is done on the assumption that the series used in the estimation is stationary while the distribution of its residual is pre-defined as a conditional Gaussian distribution. Meanwhile, the latter assumption gives us hints on how to check the adequacy of the established MGARCH model.
Results and Discussion

Figure 4.1 Time plot of the raw data Exchange rate and crude oil prices against the years from 1991-2017
Figure 4.3 Time plot of the returns on exchange rate and crude oil prices against the years from 1991-2017

Table 4.1 descriptive statistics on the returns on exchange rate and crude oil prices

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
</tr>
</thead>
<tbody>
<tr>
<td>REXCHRATES</td>
<td>1.0860</td>
<td>0.2011</td>
<td>20.5376</td>
<td>-15.0073</td>
<td>4.1486</td>
<td>0.943094</td>
<td>8.976191</td>
<td>528.5433</td>
</tr>
<tr>
<td>RCROILPRICES</td>
<td>0.3020</td>
<td>0.78798</td>
<td>21.1575</td>
<td>-32.1046</td>
<td>8.6994</td>
<td>-0.679</td>
<td>3.9804</td>
<td>37.7899</td>
</tr>
</tbody>
</table>

Source: Researcher’s computations, 2019 using Eviews version22
Table 4.2: Correlation between Return on Prices Series

<table>
<thead>
<tr>
<th>Prices Series</th>
<th>Exchrates</th>
<th>Croil prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchrates</td>
<td>1.000000</td>
<td>-0.060738</td>
</tr>
<tr>
<td>Croil prices</td>
<td>-0.060738</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

*Source: Researcher’s computations, 2019 using Eviews version22*

Table 4.3: VAR Lag Order Selection Criteria

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2047.327</td>
<td>NA</td>
<td>1303.345</td>
<td>12.84844</td>
<td>12.87205</td>
<td>12.85787</td>
</tr>
<tr>
<td>3</td>
<td>-2008.945</td>
<td>5.398813</td>
<td>1104.667</td>
<td>12.68304</td>
<td>12.84828</td>
<td>12.74903</td>
</tr>
</tbody>
</table>

* indicates lag order selected by the criterion

*Source: Researcher’s computations, 2019 using Eviews version22*

Table 4.4: Test for Cointegration

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized Cointegration Rank Test (Trace)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of CE(s)</td>
</tr>
<tr>
<td>None</td>
</tr>
<tr>
<td>At most 1</td>
</tr>
</tbody>
</table>

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level

* indicates rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized Cointegration Rank Test (Maximum Eigenvalue)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of CE(s)</td>
</tr>
<tr>
<td>None</td>
</tr>
<tr>
<td>At most 1</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level

* indicates rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

*Source: Researcher’s computations, 2019 using Eviews version22*

Table 4.5: RESULTS OF ERROR CORRECTION MODEL

<table>
<thead>
<tr>
<th>Prices Series</th>
<th>Cointegration Rank</th>
<th>ECT</th>
<th>Wald Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>RExchrates</td>
<td>1</td>
<td>-0.314107((0.05632))</td>
<td>1.125(0.045)</td>
</tr>
<tr>
<td>RCroil prices</td>
<td>1</td>
<td>-0.627834((0.10928))</td>
<td>1295(0.059)</td>
</tr>
</tbody>
</table>

*Source: Researcher’s computations, 2019 using Eviews version22*
Table 4.6: Test Roots of Characteristic Polynomial

<table>
<thead>
<tr>
<th>Endogenous variables: RECHRATES</th>
<th>RCROILPRICES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous variables: C</td>
<td>C</td>
</tr>
<tr>
<td>Lag specification: 1 2</td>
<td>Lag specification: 1 2</td>
</tr>
<tr>
<td>Date: 12/21/18   Time: 22:07</td>
<td>Date: 12/21/18   Time: 22:07</td>
</tr>
<tr>
<td>Root</td>
<td>Modulus</td>
</tr>
<tr>
<td>0.358459 - 0.271733i</td>
<td>0.449813</td>
</tr>
<tr>
<td>0.358459 + 0.271733i</td>
<td>0.449813</td>
</tr>
<tr>
<td>-0.096195 - 0.274848i</td>
<td>0.291196</td>
</tr>
<tr>
<td>-0.096195 + 0.274848i</td>
<td>0.291196</td>
</tr>
</tbody>
</table>

No root lies outside the unit circle.
VAR satisfies the stability condition.

Source: Researcher’s computations, 2019 using Eviews version 2.

![Inverse Roots of AR Characteristic Polynomial](image)

Figure 1

Table 4.7 VAR Residual Heteroskedasticity Tests (Includes Cross Terms)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>res1*res1</td>
<td>0.312858</td>
<td>9.951642</td>
<td>0.0000</td>
<td>100.4275</td>
<td>0.0000</td>
</tr>
<tr>
<td>res2*res2</td>
<td>0.101590</td>
<td>2.471546</td>
<td>0.0025</td>
<td>32.61032</td>
<td>0.0033</td>
</tr>
<tr>
<td>res2*res1</td>
<td>0.079928</td>
<td>1.898768</td>
<td>0.0260</td>
<td>25.65696</td>
<td>0.0286</td>
</tr>
</tbody>
</table>

Source: Researcher’s computations, 2019 using Eviews version 2.
The Estimated Parameters of univariate and Bivariate GARCH (DVECH-GARCH, DIAGONAL BEKK-GARCH and DCC-GARCH Model)

Estimation of the Univariate Model GARCH (1,1)

Mean: \( \text{Rcroilprices}^2 = 0.268 + 0.326 \varepsilon_t \) (4.11)

Variance Equation: \( \text{Rcroilprices}^2 = 0.975 + 0.266 \varepsilon_{t-1}^2 + 0.6826 \hat{\sigma}_{t-1}^2 \) (4.12)

Volatility persistence \((\alpha + B) = 0.266 + 0.682 = 0.948\)

Similarly, the GARCH (1,1) equation for returns on crude oil prices can be presented as thus:

Mean: \( \text{Rcroilprices} = 0.166 + 0.140 \varepsilon_t \) (4.13)

Variance: \( \text{Rcroilprices}^2 = 4.055 + 0.165 \varepsilon_{t-1}^2 + 0.791 \hat{\sigma}_{t-1}^2 \) (4.14)

Volatility persistent \((\alpha + B) = 0.165+0.791= 0.956\)

Similarly, diagonal VECH – GARCH

Estimated can re represented as thus

\[
\begin{align*}
\text{Mean component} &= \begin{bmatrix} 0.252 \\ 0.163 \\ 0.273 \\ 0.552 \end{bmatrix} \\
\text{Variance - covariance component of the model} \\
\hat{\delta}_{11,t} &= 1.774 + 0.513 \varepsilon_{t-1}^2 + 0.459 \hat{\sigma}_{11,t-1}^2 \\
(0.000) & (0.000) (0.000) \\
\hat{\sigma}_{22,t} &= 4.268 + 0.186 \varepsilon_{t-1}^2 + 0.770 \hat{\sigma}_{22,t}^2 \\
(0.045) & (0.000) (0.000) \\
\text{COV} &= -3.042 + 0.082 \hat{\sigma}_{1}, \hat{\sigma}_{2}, \hat{\sigma}_{1}, \hat{\sigma}_{2,t-1} \end{align*}
\] (4.16)

Also the diagonal BEKK – GARCH model estimated can also be represented as thus:

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\begin{align*}
\text{mean component} : - \begin{bmatrix} C(1) = 0.200 \\ C(2) = 0.517 \end{bmatrix} \\
\text{Variance – Covariance component of the model:} \\
\hat{\sigma}_{11,t} &= 1.743 + (0.704)^2 \varepsilon_{t-1}^2 + (0.692)^2 \sigma_{11,t-1}^2 \\
(0.000) & (0.000) (0.000) \\
\end{align*}
\] (4.20)
\[ \hat{\sigma}_{22t} = 9.886 + (0.275)^2 \epsilon_2^2 t_{t-1} + (0.891)^2 \sigma_{22,t-1}^2 \]  
\[ (0.0867) \quad (0.0000) \quad (0.0000) \]  
\[ COV = -1.012 + (0.704)^2 + (0.275)^2 \epsilon_{-1}^2 \epsilon_{2t} - 1 + (0.697)^2 + (0.891)^2 \sigma_{11,t-1} \]  
\[ (0.0000 + (0.000)(0.0782) \quad (0.0000 + 0.0000) \]

Similarly, the estimated Bivariate Dynamic constant conditional correlation model can be represented as thus:

\[ C(1) = \begin{cases} 
0.257 \\
0.155 
\end{cases} \]  
\[ C(2) = \begin{cases} 
0.318 \\
0.489 
\end{cases} \]  

Variance – covariance component of the model

\[ \hat{\sigma}_{11t} = 1.803 + 0.534 \epsilon_{1t-1}^2 + 0.446 \sigma_{11,t-1}^2 \]  
\[ (0.000) \quad (0.000) \quad (0.000) \]

\[ \hat{\sigma}_{12t} = 4.602 + 0.186 \epsilon_{2t-1}^2 + 0.763 \sigma_{22,t-1}^2 \]  
\[ (0.059) \quad (0.000) \quad (0.000) \]

\[ COV = -0.120 \quad SQRT (\sigma_{11,t-1}^2 \cdot \sigma_{22,t-1}^2) \]  
\[ (0.0778) \]

**Discussion of Results**

In figure 1 and 2 above illustrates the dynamics of the movement of the exchange rate and crude oil price series. Figure 1 reveal an upward trend with a sharp decline in December 2008 which later shows an upward trending in March, 2010 and another sharp decline in January 2016. Similarly, figure 2 reveals an upward trending across the period under investigation with a sharp projection in January 2017. However, the graph could be likened to a point moving with reference to time, in some cases, the movement seems to be linear (see figure 2) while the rise and falls in the trend reflects to the propensity to sharp practices in the markets, oil theft, crises or economic recession. Figure 3 and 4 above, clearly show evidence of volatility clustering in the returns series of both exchange rate and crude oil prices (US dollars/Barrel). The time plot reveals that the volatility of the two variables were not kept constant and this evidence confirmed that high returns on the series tends to followed by their corresponding high returns and low returns tend to be closed with low returns. The simple descriptive statistics of the returns on each of the series have been reported in table (4.11). The results in table (4.1) shows that all the variables used in the study have positive mean signs, meaning that the variables exhibits the characteristics of mean deviating while the standard deviation measures the degree of risks associated with the returns on each of the series. For returns on exchange rate (rexchrate), the standard deviation was (4.1485) and the returns on crude oil prices (8.694402), similarly, the co-efficient of skewness for rexchrate (0.943094) which means it is positively skewed to the right while Rcrude oil price (-0.67946) less than zero indicating that the distribution is negatively Skewed to the left which is one of the common characteristics of fairness in crude oil price return series. Also, the Co-efficient of the kurtosis for rexchrate (8.976191) is greater than three (3) and for rcrudeoil prices (3.980441) which is also greater
than three (3). This simply means the variables are not normally distributed. However, the Jarque-Bera values for rexchrate were (528.5433) and rcrude oil price (37.78986). This shows that none of the variables exhibit normal distribution characteristics. Table (4.2) shows the correlation between returns on exchange rate (rexchrate) and crude oil price (rcrude oil price) to be very low (-0.091291) implying low on movement and integration between the two markets. Tables (4.3) reveals that the lag of order four (4) is sufficient for model selection based on Akaike information criteria (AIC). However, the LR tested statistic (LR), Final predictor Error (FPE), select the same Lag length with the AIC except Schwartz Bayesian information criterion (SBC) and Hannan- Chinn information criterion (HQ) that selected Lag 2 and 3 respectively. Also, table 4.3 represents the Johansan cointegration test for rexchrates and rcrudeoilprices. In these results, the trace test statistic revealed that null hypothesis of no cointegration should be rejected but failing to reject the alternative hypothesis. This result suggests that there exist a long run relationship between the returns on exchange rates (rexchrates) and crude oil prices (rcrudeoil prices). It means the two variables can be combining in a linear fashion such that the Shocks in their short may affect movement of the individual series, which shows that the series can converge with time (in the long run).

Table (4.4) contains the results of the vector correction model and the result shows that the coefficient of error correction (-0.314107) in exchange rate markets fulfill negative condition and it is also significant. This simply means that about 31.4107% of deviations into disequilibrium between the returns on exchange rate and crude oil prices are quickly adjusted within a period (one month). This was also confirmed with a t-statistic as (-5.57722) and it’s statistically significant. The result shows that there is presence of short run causality is not significant in the two directions. This simply means that exchange rate and crude oil market prices do not govern each other in short run. VAR stability conditional test was done as a routine check to verify the dynamic stability in the movement of the two variables and the position of the inverse roots, as reflected in the figure 4.6. As it is reveal in the above figure, the roots are all inside the circle thereby confirming that the model is dynamical stable. Table (4.7) contains the results for the test for the presence of heteroscedasticity in the residuals of the fitted error correction model.

The individual market series volatility pattern was also estimated using univariate GRACH model and the results reveal the existence of strong condition for evolution of volatility. The addition of the ARCH (α) and GARCH (β) one less one justifying the condition of stationarity (model 4.12 and 4.14). The results clearly mean that past volatility rexchrate greater influence on actual volatility than past shocks in two cases. The model (4.15 – 4.18) reveal the results of the bivariate GARCH using diagonal VECH model. The estimate of (2),the unconditional mean of the covariance between returns on exchange rate and crude oil prices is positive but is not significant. This seems to contradict the observation of a negative correlation between exchange rate and crude oil prices return in Duong’s (2017) results. Also, these results contradict Duong’s (2017) findings in his investigation about the relationship between crude oil process and the US. Dollar exchange rates: constant or time–varying? In Douong (2017), it was found that the unconditional mean of the covariance between crude oil and exchange rate returns have negative sign and it was significant at the 0.05 levels of significance. The ARCH (A(1,2))and GARCH (B(2,2)) terms in the covariance model it was also confirm that both positive signs. Meanwhile in this present study, ARCH term is not significant while the GARCH term is significant at the 0.01 level meaning that the covariance between exchange rate and crude oil prices may likely cluster with respect to time. For the estimate of A(1,2) to have positive sign, it means that shocks of
exchange rate and crude oil prices of the sign may influence the conditional covariance positively, whereas shocks of the opposite signs influence covariance forecast negatively. Clearly, two estimates having negative (or positive) shocks may leads to a significant increase in next periods covariance. Also, having the unconditional mean of the covariance, A(2) not to be significantly negative, two shocks of the same sign would not likely decrease and two shocks of opposite signs would not also increase the predicted covariance in absolute value terms as against Duonge (2017) findings in examining the relationship between crude oil prices and the U.S Dollar exchange rates: constant or time-varying?.

In like manner, a non-significant positive estimate of ARCH term also confirmed that causes of the correlation between the returns on exchange rate and crude oil prices value tend not to persist. However, using the diagonal VECH model to examine the time variability in the covariance of exchange rate and crude oil prices is mainly done due to variation in the two of variance of both markets.

The results on fitted diagonal BEKK-GARCH model estimates are shown in model (4.19 - 4.22) respectively. Almost all the co-efficient are significant. For the returns on crude oil price series, the estimated GARCH parameters are evidently larger than their corresponding ARCH coefficients (whose value falls within the range: -1.012 to 0.69) with the lagged advancing the facts that the variances of these returns on exchange rate and crude oil prices are more disposed by their own lagged values, rather than “present news” which are shown by the lagged advancement. All the coefficients of the estimated parameters in the two variables shows that shock in all the markets will affect the covariance in a positive manner. The spread of volatility from returns on exchange rate to crude oil prices markets 0.697 which imply 1% increases in the returns on exchange rate market spread 69.7% volatility to crude oil markets. Conversely, the spread of volatility from crude oil market to exchange rate markets was estimated in like manner, the results of Dynamics Conditional Correlation (DCC) model is shown in model (4.23-24.25) respectively.

The result obtained in this model reflects the dynamic pattern of the dependence or the effect of volatility of returns on price of exchange rate and crude oil. The co-efficient of the GARCH (1,1) parameters are highly significant suggesting time varying variance-covariance model also reveal valid reason to use bivariate GARCH modeling for exchange rate and crude oil market. The impact of volatility is achieved by (ARCH+GARCH) which was reveal to be less than one, therefore the unconditional variance is finite. The (DCC(β)) in the conditional correlation is relatively small, having positive sign and also significant.

Similarly, the estimated GARCH parameters (DCC(β)) is comparatively large which reveal a high time varying correlations with a high degree of impact. The dynamics conditional correlation (DCC) results obtained here reveal the existence of time varying and dynamic correlation between exchange rate return crude oil prices returns.

In another development, the estimated models were considered on the bases of the model with the least Akaike information criterion information was diagonal VECH-GARCH. Similarly, model validity was also tested using all tests for serial correlation of the probability integral transforms and it was found to be noteworthy.

**Conclusion**

This study examined the dynamics associated with returns on markets prices, for example, exchange rate and crude oil prices. The study tried to explore the existence of volatility and the role it plays in two notable foreign market operations. However, in a univariate returns, it is often represented in the form of conditional variance or standard deviations. According to Wong and Yao (2005), many statistical models have been developed to capture covariance
conditional variance process. But the univariate volatility models have been found to have some limitations for example they can only model conditional variance of a single series independently among several other series. Therefore, limitations associated with it could be attributed to two basic reasons. Firstly, unvaried vitality cannot capture “Volatility spillovers” between markets or stocks returns, the unwarranted model can lead to misspecification.

The contribution this study makes is to provide strong evidence to show that the relationship between returns on exchange rate and crude oil prices is in the opposite directional movement. In Duong (2017) investigation of the relationship between crude oil prices and the U.S Dollar exchange rates; constant or time- varying, it was revealed that the unconditional mean of the covariance between crude oil exchange rate return was negative.

This study model unconditional mean of the covariance between returns on exchange rate and crude oil prices which it has been confirmed to be positive and non-significant, which is directly in the opposite of the former. This is to confirm that there exist an inverse relative between the two variables. Apparently one negative (or positive) shock in one market may lead either to appreciation or depreciation in the other market.

Secondly, the covariances between the two market series as well as its corresponding variance cannot be captured with univariate models. Therefore, all the challenges mentioned above that cannot be captured with univariate model requires covariances as inputs; therefore, this study used multivariate GARCH models which can potentially captures all models. Thus, this study attempt to explore volatility spillover in diagonal VECH, BECKK GARCH and DCC model volatility in each of the foreign markets. Also, the independents as well as volatility spillover between the returns on exchange rate and crude oil prices market was also confirmed. However, the volatility spillover observed in the model estimation was found to be two directions as there was strong negative inter-relation between the two markets.

Reference
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